

(3 Hours)

[Total marks : 80]

Note :-

- 1) Question number 1 is **compulsory**.
- 2) Attempt any **three** questions from the remaining **five** questions.
- 3) **Figures to the right** indicate **full marks**.

- Q.1** a) Find the Laplace transform of $\cos t \cos 2t \cos 3t$. 05
- b) Construct an analytic function whose real part is $e^x \cos y$. 05
- c) Find the directional derivative of $\phi = x^4 + y^4 + z^4$ at point A (1, -2, 1) in the direction of AB where B is (2, 6, -1). 05
- d) Expand $f(x) = lx - x^2$, $0 < x < l$ in a half-range sine-series. 05
- Q.2** a) Find the angle between the normals to the surface $xy = z^2$ at the points (1, 4, 2), (-3, -3, 3). 06
- b) Find the Fourier series for

$$f(x) = \begin{cases} -c & -a < x < 0 \\ c, & 0 < x < a \end{cases}$$
 06
- c) Find the inverse Laplace transform of 08
- (i)
$$\frac{4s + 12}{s^2 + 8s + 12}$$
- (ii)
$$\log\left(\frac{s^2 + a^2}{\sqrt{s + b}}\right)$$
- Q.3** a) State true or false with proper justification "There does not exist an analytic function whose real part is $x^3 - 3x^2y - y^3$ ". 06
- b) Prove that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right)$. 06
- c) Expand $f(x) = 4 - x^2$ in the interval (0, 2). 08
- Q.4** a) Use Gauss's Divergence theorem to evaluate $\iint_S \bar{N} \cdot \bar{F} dS$ where $\bar{F} = 4x i + 3y j - 2z k$ and S is the surface bounded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$. 06

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- b) Prove that $\int x^3 \cdot J_0(x) dx = x^3 \cdot J_1(x) - 2x^2 \cdot J_2(x)$. 06
- c) Solve using Laplace transform $\frac{dy}{dt} + 3y = 2 + e^{-t}$ with $y(0) = 1$. 08
- Q. 5 a) Find Laplace transform of $(1 + 2t - 3t^2 + 4t^3)H(t - 2)$ where 06
 $H(t - 2) = \begin{cases} 0, & t < 2 \\ 1, & t \geq 2 \end{cases}$
- b) Prove that $2J_0''(x) = J_2(x) - J_0(x)$. 06
- c) Obtain complex form of Fourier Series for $f(x) = e^{ax}$ in $(-\pi, \pi)$ 08
where a is not an integer. Hence deduce that when a is a constant other than an integer
- $$\sin ax = \frac{\sin \pi a}{i\pi} \sum \frac{(-1)^n n}{(\alpha^2 - n^2)} e^{inx}$$
- Q. 6 a) Using Green's theorem evaluate 06

$$\oint_C (e^{x^2} - xy) dx - (y^2 - ax) dy$$

where C is the circle $x^2 + y^2 = a^2$.
- b) Express the function 06
 $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$
as a Fourier Integral.
- c) Under the transformation $w = (1+i)z + (2-i)$, find the region 08
in the w -plane into which the rectangular region bounded by
 $x = 0, y = 0, x = 1, y = 2$ in the z -plane is mapped.

xxx

Circuits & Transmission Lines .

Q.P. Code: 24590

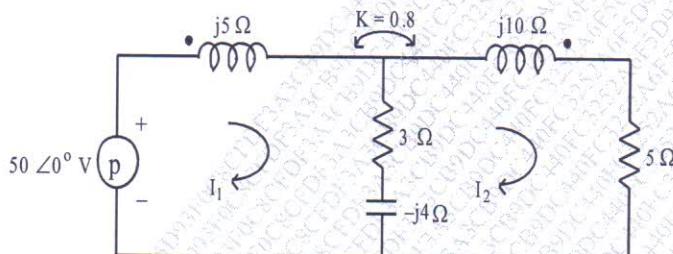
Time: 3 hours

Total Marks: 80

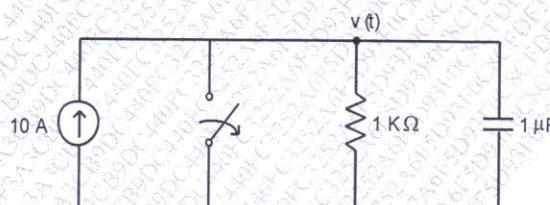
N.B.

- 1) Question No. 1 is Compulsory
- 2) Out of remaining questions, attempt any three
- 3) Assume suitable data if required
- 4) Figures to the right indicate full marks

I (A) Draw equivalent circuit for given magnetically coupled circuit. 05



(B) In the given network of Fig., switch is opened at $t = 0$. Solve for v and $\frac{dv}{dt}$ at $t = 0+$. 05

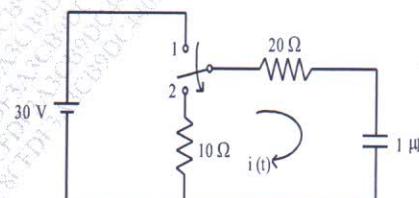


(C) Prove that $AD - BC = 1$ for Transmission parameters. 05

(D) Define the following parameter of transmission lines: 05

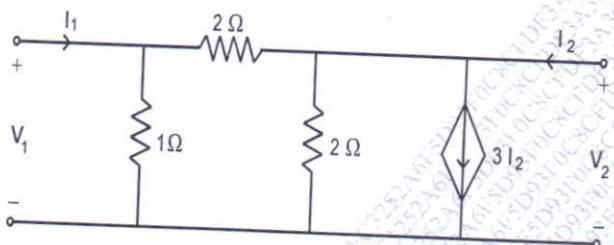
- i) Input impedance
- ii) Characteristics Impedance
- iii) VSWR
- iv) Reflection Coefficient
- v) Transmission Coefficient

2 (A) In the network shown in Fig., switch is changed from position 1 to position 2 at $t = 0$, steady condition having reached before switching. Find the values of i , $\frac{di}{dt}$ and $\frac{d^2i}{dt^2}$ at $t = 0+$. 10



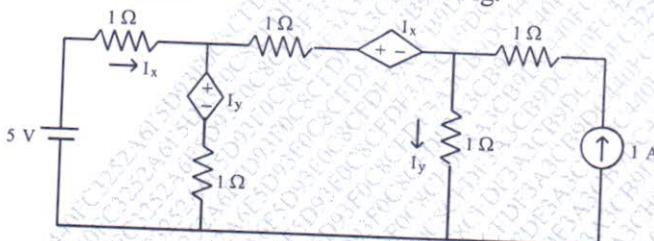
- (B) For the network shown in Fig., find Z and Y-parameters.

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- 3 (A) Find currents in the three meshes of network shown in Fig.

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- (B) The parameters of a transmission lines are $R = 65\Omega/\text{km}$, $L = 1.6\text{mH}/\text{km}$, $G = 2.25 \text{ mmho/km}$, $C = 0.1\mu\text{F}/\text{km}$. Find
 i) Characteristic Impedance
 ii) Propagation Constant
 iii) Attenuation Constant
 iv) Phase Constant at 1 kHz

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- 4 (A) Determine whether following functions are positive real

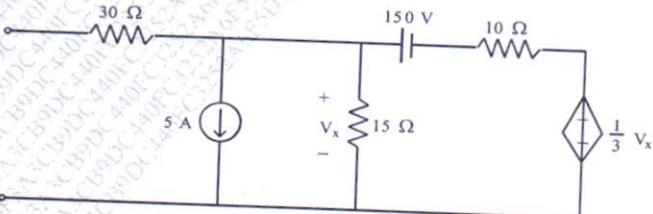
$$\text{i) } \frac{s^4 + 3s^3 + s^2 + s + 2}{s^3 + s^2 + s + 1}$$

$$\text{ii) } \frac{s(s+3)(s+5)}{(s+1)(s+4)}$$

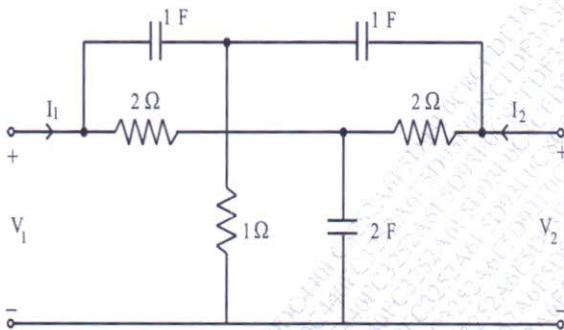
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- (B) Obtain Thevenin equivalent network of Fig.

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- 5 (A) Find Y-parameters for the network shown in Fig.



- (B) Realize the following functions in Foster II and Cauer I form

$$Z(s) = \frac{2(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)}$$

- 6 (A) A transmission line has a characteristics impedance of 50 ohm and terminate in a load $Z_L = 25 + j50$ ohm. Use smith chart and Find VSWR and Reflection coefficient at the load.

- (B) In the network of Fig. switch is in position 'a' for a long time. At $t = 0$ switch is moved from a to b. Find $v_2(t)$. Assume that the initial current in 2 H inductor is zero.

